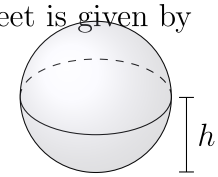
Here is an application of the chain rule.

(1) The volume of water, in cubic feet, in a spherical tank of radius 4 feet is given by

$$V = \frac{\pi}{3}h^2(12 - h),$$

where h is the height, in feet, of the water in the tank.



Water flows in and out of the tank regularly. The height of the water in the tank is given by

$$h = \sin(\pi t) + 2,$$

where t is time, in hours elapsed since some starting time.

(a) Find the instantaneous rate of change (with correct units) of the volume with respect to *height* when $t = 1$.

(b) Find the instantaneous rate of change (with correct units) of the volume with respect to *time* when $t = 1$.

(2) A simple model of the Lion population in the Kruger national park in South Africa is as follows:

$$L(s) = 100\sin(\frac{\pi}{6}s^2 - \frac{\pi}{6}s + \pi/3) + 1000$$

Where L is the number of lions and s is the number of *springbok*[1]. The population of springbok is given by

$$s(g) = 500\cos(\frac{\pi}{6}g + \frac{\pi}{4}) + 10000$$

where g is the total available grazing biomass, measured in tonnes.

(a) Find the instantaneous rate of change of the lion population with respect to the springbok population when $g = 100000$.

(b) Find the instantaneous change of the lion population with respect to the total available grazing biomass when $g = 100000$. What are the correct units for this quantity?

[1] A springbok is a small antelope favoured by lions