Math 151BH: Honors Numerical Analysis Part 2

Course Description: This course is a rigorous introduction to the study of numerical algorithms. Students will be equipped with the skills necessary to apply these algorithms in a principled manner to problems arising in statistics, imaging, data science, engineering and related fields. We focus on four fundamental problems: finding eigenvalues, finding numerical solutions to ordinary differential equations (ODE's), the least squares problem and the fast Fourier transform.

Instructor: Daniel Mckenzie, `mckenzie@math.ucla.edu`.

Teaching Assistant: Cameron Kissler, `ckissler@math.ucla.edu`.

Office Hours:

- Daniel Mckenzie: TBD.
- Cameron Kissler: TBD.

Course Format: Three one-hour lectures and one one-hour discussion section per week. The lectures are MWF 10:00–10:50 AM in Zoom Room: `https://ucla.zoom.us/j/97362667580`. The discussion section is T 10:00–10:50 AM.

Prerequisites: Math 32B, Math 33B, Math 115A, Math 131A, Math 151AH and either PIC 10A or CS 31. Math 151A can be substituted for Math 151AH, provided you obtained a B or higher.

Learning Outcomes: By the end of the course, students will:

1. Recognize eigenvalue problems in real-world applications. Understand various schemes for finding eigenvalue/eigenvector pairs, such as the power method and Lanczos method. Know when various methods are appropriate.
2. Understand the existence and uniqueness for ordinary differential equations (ODE's). Be able to apply simple ODE solvers (such as Euler's method) as well as more sophisticated methods (such as Runge-Kutta or Predictor-Corrector) to initial value problems. Understand the difference between in initial value problem and a boundary problem. Be able to apply shooting methods and finite difference methods to boundary value problems.
3. Appreciate the importance of the least squares problem. Be able to solve least squares problems using QR factorization, conjugate gradient method, gradient descent and Kaszmarz method.
4. Realize the ubiquity of the Fourier transform in modern communication and computation systems. Understand and be able to implement the Fast Fourier Transform.

Communication: This class will use Piazza for classroom discussion. If you have questions about the course content (*e.g.* "Where do I start with question 14 on the homework?" or "Can someone explain that thing we did at the end of class yesterday?") or of an administrative nature (*e.g.* "When is the second midterm?"), post them here. The sign up link is `https://piazza.com/ucla/spring2021/math151bh/home`. We (*i.e.* your instructor and T.A.) will be checking Piazza regularly, and will try our absolute best to ensure that all questions are answered within 24 hours. If you have questions about your grades, or other questions/comments of a personal nature, you should contact me or your T.A. via email. Office hours or immediately after class are also good times to ask questions.

Textbooks: *Numerical Analysis* by L. Ridgeway Scott, Princeton University Press (LRS). *Numerical Analysis* 10th Ed. by R. Burden and J. Faires (BF). One section (QR decomposition) will be based on a chapter from *Matrix Methods in Data Mining and Pattern Recognition* by Lars Elden (E). It may be useful to consult this book for this section, but it is not essential.

Proposed Schedule		
Week	**Reading Assignment**	**Content**
1.	Chpt. 8.1–8.2 (BF) and Chpt. 4–5 (E)	Brief review of linear algebra. The least squares problem. QR decompositions, Householder transformations.
2.	Chpt. 8.1–8.2 (BF) and Chpt. 9 (LRS)	The conjugate gradient method. The Kacsmarz method. Ridge regression and LASSO.
3.	Chpt. 14 (LRS)	Introduction to eigenvalue problems. Some sample applications. Gershgorin's disks. Finding all vs. finding highest eigenvalue. Power method. Hessenberg factorizations and finding all eigenvalues.
4.	Chpt. 15 (LRS)	Eigenvalue algorithms. Power method, inverse iteration and deflation. Singular Value Decomposition. Finding all eigenvalues using QR decomposition and using Jacobi iteration.
5.	Chpt. 16 (LRS) and Chpt. 5.9 (BF)	Ordinary differential equations. Existence and uniqueness of solutions. Euler and implicit Euler methods. Error estimates.
6.	Chpt. 17 (LRS) and Chpt 5.4 (BF)	Systems of differential equations and higher order differential equations. Higher order solvers for initial value problems. Runge-Kutta.
7.	Chpt 5.6, 5.10 and 5.11 (BF)	Stability for numerical ODE solvers. Implicit schemes such as Adams-Moulton. Multi-step and predictor corrector schemes. Stability.
8.	Chpt. 11.1–11.4 (BF)	Boundary value problems. Linear and nonlinear shooting methods. Finite difference methods.
9.	Chpt. 8.5 and 8.6 (BF)	Trigonometric polynomial approximation. Elementary Fourier theory. The fast Fourier transform.
10.		Review and catch-up.

Grading: Your grade will be determined using the following formula: 45% Homework + 25% Midterm + 30% Final exam. You may earn up to 2% extra credit by asking and answering questions on Piazza (see: "Communication"). Grades will be assigned using the following grade lines. A: $\geq$ 93%, A-: $\geq$ 90%, B+ $\geq$ 87%, B: $\geq$ 83%, B- $\geq$ 80%, C+ $\geq$ 77%, C: $\geq$ 73%, C-: $\geq$ 70%, D+: $\geq$ 67%, D: $\geq$ 63%, D- $\geq$ 60%, F: $<$ 60%.

Homework: There will be four homework assignments for this course, with due dates:

- Homework 1: Friday, April 16th.
- Homework 2: Friday, April 30th.
- Homework 3: Friday, May 14th.
- Homework 4: Friday, May 28th.

You are to submit your homework using GradeScope. We strongly encourage you to use word processing software such as LaTeX to type your solutions. The penalty for late homework is 10% of your score per day after the due date. Homework problems will be a mix of pen-and-paper calculations and programming exercises. You are encouraged to discuss homework questions on Piazza or in person. However copying other's solutions or programs is considered a serious violation, and you should never share your written solutions with anyone.

Software: You may use any language you like, but Python and MATLAB are highly recommended. As a UCLA student you can get a licensed version for free at `https://softwarecentral.ucla.edu/matlab-getmatlab`.

Exams: There will be one in-class midterm exam and a non-cumulative final. For both, you will have a 24 hour window to complete and submit the exam. Make-ups for the final and midterm are permitted only under exceptional circumstances, as outlined in the UCLA student handbook. **Note that you must take the final exam in order to pass this class.** The exams are scheduled for the following dates:

- Midterm: Monday, May 10th.
- Final Exam: Monday, June 7th.

Note that both exams will be of the pen-and-paper variety, but you may be asked to write pseudocode or implement, by hand, a few steps of an algorithm on a toy example.

Classroom Expectations

- Slides for all lectures will be posted on Piazza 24 hours before the lecture. I encourage you to download them before class, so that you can skim over them, and so that you can add your own annotations to your copy during class. All lectures will be recorded and available on CCLE. You are expected to attend lectures and discussion sections if you are able. If you are in a different time zones, the expectation is that you watch the recordings of all lectures and discussion sections.

- On exams I expect you to give legible, well-justified solutions. A numerical answer without any supporting calculations is unlikely to receive credit, nor is a solution that I can't read.
- Standard Zoom etiquette applies. For example, mute your mic unless you have a question. You are encouraged, but not required to turn your cameras on.
- In this classroom you can expect to be treated with respect, regardless of your age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation and other visible and non-visible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

How to succeed in this course

- Prepare for class by by looking over the slides or reading ahead in the textbook. Spending even 15 minutes skimming a section before it is taught in class can be useful!
- Office hours are time that we (your instructor and T.A.'s) set aside in order to discuss math with you. You can drop in at any time during office hours. They are a great time to discuss: homework problems you are stuck on, things you didn't quite understand in class, strategies for preparing for or taking tests or going over tests after they have been returned. We may also be able to help you debug your code
- Forming a study group can be very helpful. Remember that often the best way to fully understand a concept is to try to explain it to someone else.
- Make use of Piazza! (See "Communication" section). Again, answering other students questions is a great way to make sure you really understand a concept.

Academic Integrity: From the office of the Dean of Students:

> *With its status as a world-class research institution, it is critical that the University uphold the highest standards of integrity both inside and outside the classroom. As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. Accordingly, when accusations of academic dishonesty occur, The Office of the Dean of Students is charged with investigating and adjudicating suspected violations. Academic dishonesty, includes, but is not limited to, cheating, fabrication, plagiarism, multiple submissions or facilitating academic misconduct.*

Students are expected to be aware of the University policy on academic integrity in the UCLA Student Conduct Code*. In particular, please note the sections on (1) cheating, (2) plagiarism, and (3) unauthorized study aids.

Accessibility: Students needing academic accommodations based on a disability should contact the Center for Accessible Education (CAE) at (310)825-1501 or in person at Murphy Hall A255. In order to ensure accommodations, students need to contact the CAE within the first two weeks of term.

*http://www.deanofstudents.ucla.edu/Portals/16/Documents/UCLACodeOfConduct_Rev030416.pdf

Notice about sexual harassment, discrimination and assault: Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault and stalking. Students who have experienced sexual harassment or sexual violence can receive **confidential** support and advocacy from a CARE advocate:

The CARE Advocacy Office for Sexual and Gender-Based Violence
1st Floor, Wooden Center West
CAREadvocate@caps.ucla.edu
(310) 206-2465

You can also report sexual violence or sexual harassment directly to the University's Title IX Coordinator:

Kathleen Salvaty
2241 Murphy Hall
titleix@conet.ucla.edu
(310) 206-3417