Instructor: Daniel Mckenzie, mckenzie@math.ucla.edu.

Teaching Assistant: Bumsu Kim, bumsu@math.ucla.edu.

Office Hours:

- Daniel Mckenzie: TBD.
- Bumsu Kim: TBD.

Textbook: Numerical Analysis by L. Ridgeway Scott, second edition.

Course Description: This course is a rigorous introduction to the study of numerical algorithms. Students will be equipped with the skills necessary to apply these algorithms in a principled manner to problems arising in statistics, imaging, data science, engineering and related fields. We focus on five fundamental problems: root-finding, solving linear systems, interpolation, quadrature and finding eigenvalues. P/NP or letter grading.

Course Format: Three one-hour lectures and one one-hour discussion section per week.

Prerequisites: Math 115A, Math 131A. Some experience in a programming language is **strongly recommended**.

Learning Outcomes: By the end of the course, students will:

- 1. Understand the implications of finite precision arithmetic for computation.
- 2. Recognize root-finding and optimization problems as special cases of the more general fixedpoint problem, and be able to formulate appropriate algorithms for both.
- 3. Understand multiple approaches to the numerical solution of linear systems and know when to apply which method.
- 4. Recognize structured linear systems (banded matrices, sparse matrices etc.) and be able to design algorithms that exploit this structure.
- 5. Appreciate the difficulty in constructing an interpolating function for a given data set. Understand the power of polynomial interpolation, as given by Weierstrass' Theorem, but also understand the limitations as given by sample complexity bounds. By able to implement various schemes, such as Lagrange polynomials and splines.
- 6. Know various quadrature rules, such as the trapezoid rule, Simpson's rule and Gaussian quadrature. Appreciate the accuracy vs. complexity trade-off present when choosing a scheme for a given application.

7. Recognize eigenvalue problems in real-world applications. Understand various schemes for finding eigenvalue/eigenvector pairs, such as the power method and Lanczos method. Know when various methods are appropriate.

Communication: This class will use Piazza for classroom discussion. If you have questions about the course content (*e.g.* "Where do I start with question 14 on the homework?" or "Can someone explain that thing we did at the end of class yesterday?") or of an administrative nature (*e.g.* "When is the second midterm?"), post them here. The sign up link is piazza.com/ucla/winter2021/ math151ah. We (*i.e.* your instructor and T.A.) will be checking Piazza regularly, and will try our absolute best to ensure that all questions are answered within 24 hours. If you have questions about your grades, or other questions/comments of a personal nature, you should contact me or your T.A. via email. Office hours or immediately after class are also good times to ask questions.

Schedule		
Week	Reading Assignment	Content
1	Chpt. 1 and Chpt. 18	Introduction to finite precision arithmetic and algorithms. Conver- gence and Stability. Floating point numbers, their arithmetic and errors. Big "O" notation.
2.	Chpt. 2	Fixed-point algorithms. Application to root-finding. Newton's method and the secant method. Connections with optimization. Error analysis.
3.	Chpt 5. and Chpt 6.	Review of linear algebra. Vector spaces and norms. Inifinite di- mensional vector spaces. Operators and operator norms. Inner products. Powers and convergence of matrices.
4.	Chpt. 3	Basic numerical methods for linear systems. Gaussian elimination. Triangular matrices and the LU decomposition. Pivoting rules. Cholesky decomposition. Application to banded matrices.
5.	Chpt. 8	Iterative methods for linear systems. Jacobi and Gauss-Seidel methods. Convergence analysis for these algorithms. Application to sparse linear systems. Matrix splittings in general.
6.	Chpt. 7	Systems of nonlinear equations. Functional iteration. Newton's method and quasi-Newton's method. Bi-level procedures for fixed point problems.

7.	Chpt. 10 and Chpt. 11	Polynomial interpolation. Connection to linear systems. Rela- tionship between Taylor polynomials and Lagrange polynomials. Higher order interpolation schemes such as Hermite polynomials. Approximation with trigonometric series.
8.	Chpt 12.	Introduction to approximation theory. Lebesgue and Sobolev spaces of functions. Weierstrass Theorem. Bernstein polynomi- als. Splines. Connection between polynomial approximation and least squares.
9.	Chpt. 13	Numerical quadrature. "Basic" schemes such as trapezoidal and Simpson's. Gaussian quadrature. Composite schemes.
10.	Chpt. 14.	Introduction to eigenvalue problems. Some sample applications. Gershgorin's disks. Finding all vs. finding highest eigenvalue. Power method. Hessenberg factorizations and finding all eigenvalues. Re- view and catch-up

Grading: Your grade will be determined using the following formula: 45% Homework + 25% Midterm + 30% Final exam. You may earn up to 2% extra credit by asking and answering questions on Piazza (see: "Communication").

Homework: There will be four homework assignments for this course, with due dates:

- Homework 1: Friday, January 15th.
- Homework 2: Friday, January 29th.
- Homework 3: Friday, February 19th.
- Homework 4: Friday, March 5th. (earlier due to Thanksgiving break)

You are to submit your homework using GradeScope. We strongly encourage you to use word processing software such as LaTeX to type your solutions. The penalty for late homework is 10% of your score per day after the due date. Homework problems will be a mix of pen-and-paper calculations and programming exercises. You are encouraged to discuss homework questions on Piazza or in person. However copying other's solutions or programs is considered a serious violation, and you should never share your written solutions with anyone.

Software: We will be using MATLAB. It is an easy to learn and versatile scientific computing language. As a UCLA student you can get a licensed version for free at https://softwarecentral.ucla.edu/matlab-getmatlab.

Exams: There will be one in-class midterm exam and a cumulative final. For both, you will have a 24 hour window to complete and submit the exam. The midterm will contain the same amount of content as a 2 hour in-person exam. The final will contain the same amount of content as a 3

hour in-person exam. Make-ups for the final and midterm are permitted only under exceptional circumstances, as outlined in the UCLA student handbook. Note that you must take the final exam in order to pass this class. The exams are scheduled for the following dates:

- Midterm: Friday, February 5th.
- Final Exam: Monday, March 15th.

Note that both exams will be of the pen-and-paper variety, but you may be asked to write pseudocode or implement, by hand, a few steps of an algorithm on a toy example.

Classroom Expectations

- Slides for all lectures will be posted on Piazza, at least 24 hours before the lecture. I encourage you to download them before class, so that you can skim over them, and so that you can add your own annotations to your copy during class. All lectures will be recorded and available on CCLE. You are expected to attend lectures and discussion sections if you are able. If you are in a different time zones, the expectation is that you watch the recordings of all lectures and discussion sections.
- On exams I expect you to give legible, well-justified solutions. A numerical answer without any supporting calculations is unlikely to receive credit, nor is a solution that I can't read.
- Standard Zoom etiquette applies. For example, mute your mic unless you have a question. You are encouraged, but not required to turn your cameras on.
- In this classroom you can expect to be treated with respect, regardless of your age, background, beliefs, ethnicity, gender, gender identity, gender expression, national origin, religious affiliation, sexual orientation and other visible and non-visible differences. All members of this class are expected to contribute to a respectful, welcoming and inclusive environment for every other member of the class.

How to succeed in this course

- Prepare for class by by looking over the slides or reading ahead in the textbook. Spending even 15 minutes skimming a section before it is taught in class can be useful!
- Office hours are time that we (your instructor and T.A.'s) set aside in order to discuss math with you. You can drop in at any time during office hours. They are a great time to discuss: homework problems you are stuck on, things you didn't quite understand in class, strategies for preparing for or taking tests or going over tests after they have been returned. We may also be able to help you debug your code
- Forming a study group can be very helpful. Remember that often the best way to fully understand a concept is to try to explain it to someone else.
- Make use of Piazza! (See "Communication" section). Again, answering other students questions is a great way to make sure you really understand a concept.

Academic Integrity: From the office of the Dean of Students:

With its status as a world-class research institution, it is critical that the University uphold the highest standards of integrity both inside and outside the classroom. As a student and member of the UCLA community, you are expected to demonstrate integrity in all of your academic endeavors. Accordingly, when accusations of academic dishonesty occur, The Office of the Dean of Students is charged with investigating and adjudicating suspected violations. Academic dishonesty, includes, but is not limited to, cheating, fabrication, plagiarism, multiple submissions or facilitating academic misconduct.

Students are expected to be aware of the University policy on academic integrity in the UCLA Student Conduct Code^{*}. In particular, please note the sections on (1) cheating, (2) plagiarism, and (3) unauthorized study aids.

Accessibility: Students needing academic accommodations based on a disability should contact the Center for Accessible Education (CAE) at (310)825-1501 or in person at Murphy Hall A255. In order to ensure accommodations, students need to contact the CAE within the first two weeks of term.

Notice about sexual harassment, discrimination and assault: Title IX prohibits gender discrimination, including sexual harassment, domestic and dating violence, sexual assault and stalking. Students who have experienced sexual harassment or sexual violence can receive **confidential** support and advocacy from a CARE advocate:

The CARE Advocacy Office for Sexual and Gender-Based Violence 1st Floor, Wooden Center West CAREadvocate@caps.ucla.edu (310) 206-2465

You can also report sexual violence or sexual harassment directly to the University's Title IX Coordinator:

Kathleen Salvaty 2241 Murphy Hall titleix@conet.ucla.edu (310) 206-3417

^{*}http://www.deanofstudents.ucla.edu/Portals/16/Documents/UCLACodeOfConduct_Rev030416.pdf